METHODS OF ASSESSING STABILITY UNDER SEISMIC LOADING FOR GRAVITY DAMS.

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ABSTRACT

The assessment of stability of gravity dams under seismic loading can currently be completed using simple methods. The methods may include the pseudostatic seismic coefficient approach or even the response spectrum method otherwise only used for stress analysis. Due to the cyclic nature of the loading, it is clear that both methods may not lead to an accurate assessment of overturning and sliding stability under a seismic event.

This paper will address results from both the above methods as well as time history analysis using both synthetic and natural time histories. By assessing the results obtained from the parametric study, effectiveness and accuracy of each method are evaluated and conclusions made. A more suitable method of early stage stability assessment is provided with a simplified approach formulated.

1. INTRODUCTION

A critical factor in the design of gravity dams is the ability of the dam to withstand applied forces leading to sliding and overturning failures. The relevant loading conditions required to carry out the stability calculations can easily be identified under static conditions, allowing for simple evaluation. Under seismic conditions, the applied loading may be difficult to ascertain and many methods have been suggested.

It is of the utmost importance that the suitable seismic loading inputs be applied and the dynamic response of the dam to these loadings be correctly defined. The objective of this paper is to determine the variations in stability results using different established methods, and reasons for the variations. Further to this evaluation, a suitable method for early stage stability assessment is formulated and advocated for use in place of the former methods which may lack a theoretical basis.

2. POTENTIAL STABILITY ANALYSIS APPROACH

Various methods of seismic stability analysis have been established through the years. These have ranged from the simplest seismic coefficient approach to most complex nonlinear time history analysis. For this paper, three of the more common analysis methods are reviewed and compared.

2.1 Seismic coefficient stability analysis

The seismic coefficient approach attempts to simplify the time and frequency dependant seismic input motion to an equivalent pseudostatic lateral load. This method essentially multiplies the inertial properties of the dam and hydrodynamic masses by an acceleration coefficient. The source of the coefficient is somewhat unfounded as certain engineers utilise the Peak Ground Acceleration (PGA) as a basis with an additional factor to account for loading type or structural importance. Using the resulting inertial loading, a simple stability calculation may be carried out and compared to the required Factor of Safety (FoS) as per normal static calculations.
2.2 Response spectrum stability analysis

A response spectrum analysis is, in general, a linear elastic method which considers discrete mass and displacement contributions from each natural frequency which leads to the maximum seismic response of a structure. Both the response due to the dam and hydrodynamic masses are considered.

The response spectrum analysis takes into account the dynamic properties of the system by considering modal mass, damping and stiffness. In general, the analysis is carried out using an input spectrum based on pseudo-spectral accelerations. Each mode is considered individually and the modal response calculated using the relevant ordinate on the given spectrum.

The final results are commonly combined using either the square root of the sum of the squares (SRSS) method or the complete quadratic combination (CQC) method, dependant on spacing of the system frequencies. For well-spaced frequencies, the SRSS method is sufficient otherwise the CQC method is preferred. These results can be post-processed to an equivalent static result to determine the relevant stability FoS.

2.3 Time history stability analysis

Time history analysis is the most appropriate way of assessing the dynamic response of a system to any form of dynamic loading including seismic loading. The dynamic properties of the system are implicit in the equation of motion which is the basis for the solution of a time history analysis.

For linear time history analysis, the solution may be carried out in either the frequency domain, Modal time history, or in the time domain, Direct integration. Results from the analysis can be extracted at discrete time points throughout the seismic event.

It is therefore possible to calculate sliding and overturning FoS at different time steps to develop a FoS time history. This is a very powerful way to assess the behaviour, especially for stability analysis. Further design criteria can be used in addition to the standard FoS which is not necessarily relevant under cyclic loading conditions.

3. DISPLACEMENTS DUE TO SLIDING

To further assess the sliding stability of a dam, a permanent deformation approach may be considered as supported by many bodies including FEMA (2005). The question is not whether the approach may be followed but rather what acceptance criteria should be used. The allowable permanent displacements are dependent on the dam joints, grout curtains, drains and possible added water pressures in damaged sections of the base.

The permanent displacement of a dam may be calculated by assuming rigid body movement along a plane. The method was first proposed by Newmark (1965). Following this premise, should the sliding FoS fall below 1, ignoring shear stiffness of the plane as well as any damping due to friction sliding, the acceleration along the plane can be determined by taking the net base shear force divided by the associated displaced mass.

This will produce an acceleration time history graph. The second integral of acceleration is displacement. Therefore the cumulative displacement may be determined based on the acceleration time history. It is noted that the rigid body displacement will only discontinue should all the inertial forces diminish to zero. Therefore the sliding only stops once a negative acceleration reduces velocity to zero. This process is illustrated in Figure 1.
4. PARAMETRIC STUDY

The Pine Flat Dam situated in California, USA was selected for use in the parametric study. The dam is a concrete gravity dam completed in 1954. The dam was selected for this study as many publications have used the same dam and therefore the current study can be read in context with the other publications. The geometry of the dam and associated finite element model are shown in Figure 2. To minimise the effects of influence of foundation flexibility, the extent of the foundation was taken as the height of the dam (H) in depth and 1.5xH either direction of the dam body.

To enable accurate comparison, the applied base forces were determined by integrating the resultant shear and normal base stresses at discrete points along the dam. The resultant applied and restoring overturning moments were determined by taking moments around the dam toe.

4.1 Loadings

Standard loadings for the study were selected. These loadings comprised hydrostatic, uplift and hydrodynamic loading. The uplift was based on the one third rule and assumed not to change under rapid seismic loading. Post seismic stability was not carried out as this is not the aim of the paper. Hydrodynamic mass was calculated using Westergaard (1933) formulation. It is noted that the defining of the added mass can easily be adapted to theory given by Chopra (1967). For this paper, it is only necessary to maintain consistency throughout the parametric study. A loading schematic is given in Figure 3.
4.2 Sliding Resistance

For this paper, the sliding resistance was determined using Mohr Coulomb theory. Throughout the study, under seismic loading, the dam-foundation interface is assumed to have only one third of the base area able to carry a cohesion of 200 kPa with an overall friction angle of 45°. These properties are assumed to cover both peak and residual properties. The interface properties are also highly debateable with various literature giving high values of friction coefficient and as well as high direct tensile strengths including (Altarejos-Garcia et al. 2015) and (ICOLD European Club 2004). This aspect is outside the scope of the paper but is an extremely important factor in the determination of dam stability.

4.3 Seismic coefficient

A coefficient equal to the PGA was used to assess the FoS. As previously stated, there is usually some form of modification to this value due to loading type or structure importance. The subsequent results can easily be changed to include such modification, but to carry out a parametric study the coefficient is required to remain equal to the seismic PGA. The PGA of the seismic input was taken as 0.25 g. Following the analysis, a FoS of 1.41 and 1.01 was determined for overturning and sliding respectively.

4.4 Response spectrum

A 5% design spectrum was developed using SANS 10160-4:2010. This design spectrum was used to create a synthetic time history which is discussed in Section 4.5. The synthetic time history was then in turn used as input to obtain an absolute acceleration response spectrum. This spectrum was used as input to ensure direct comparison of methods.

An eigenvalue solution was performed and indicated the first and second modes at 1.94 Hz and 4.35 Hz respectively. A minimum of 90% participation is required to carry out a response spectrum analysis and this was achieved within the first four modes (see Table 1).

<table>
<thead>
<tr>
<th>Mode</th>
<th>$T_{\text{hor}}$ (%)</th>
<th>$T_{\text{vert}}$ (%)</th>
<th>(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.67%</td>
<td>1.19%</td>
<td>1.94</td>
</tr>
<tr>
<td>2</td>
<td>26.94%</td>
<td>0.19%</td>
<td>4.35</td>
</tr>
<tr>
<td>3</td>
<td>1.49%</td>
<td>90.54%</td>
<td>5.71</td>
</tr>
<tr>
<td>4</td>
<td>6.87%</td>
<td>2.73%</td>
<td>7.45</td>
</tr>
</tbody>
</table>

Following the eigen value solution a response spectrum analysis was carried out and results post-processed. The results indicated a FoS of 0.70 and 0.80 for overturning and sliding respectively.
4.5 Time history

As the frequency content of seismic events vary considerably from one event to another, three separate time histories were evaluated. These comprise a synthetic time history, the 1940 El Centro event at 180 deg and lastly the 1979 El Centro event at 220 deg as shown in Figure 4.

Figure 4. Seismic inputs – Time histories & Response spectra

The synthetic time history was created by frequency matching the synthetic spectrum to that of the SANS design spectrum which can be completed using various mathematical formulations not covered in this paper. It is however important to note that the duration of shaking is an important aspect to synthetic time history generation and for this study a duration of 30 seconds was used.

To capture the effect of damping on stability calculations, both 5% and 7% damping were used. These are common values used for concrete prior and post damage. The value of damping for dams has been a long contested subject with ongoing research being carried out. For this study, the traditional values are considered satisfactory.

The synthetic time history was developed with a PGA of 0.25 g and both the 1940 El Centro and 1979 El Centro time histories were linearly scaled to have a PGA of 0.25 g. The three spectra, shown in Figure 4, envelope a wide range of frequencies which indicate any dynamic amplification would be sufficiently captured in the analyses. Including three time history analyses will also give insight as to the change in response of the dam to differing input frequency contents but with a constant seismic PGA.

The time history analyses were carried out using the modal time history solution method. A constant direct modal damping of 5% and 7% was incorporated. FoS for overturning and sliding, Figure 5, as well as cumulative displacements, Figure 6, were calculated using techniques as described in Section 3.
4.6 Result comparison and discussion

It is clear that there is a wide array of results as can be seen in Table 2. Analysis using the seismic coefficient approach produced results acceptable for the stability of a dam under seismic events. But these results do not include certain adjustments which would normally be made for loading type or structural importance. Following the inclusion of these factors, the stability evaluation may indicate either inadequacy or safety, dependant on the applied factor.

The response spectrum analysis over predicted the dynamic response of the dam and indicated a low factor of safety when compared to the results of the SANS synthetic time history analysis. This is an expected result as the response spectrum method incorporates peak responses in its formulation. It is therefore established that this method, although adequate for stress analysis, should not be used for stability calculations.

The time history analyses produced a large discrepancy in displacement results. This is due to the dynamic properties of the dam system and differing amplitudes of frequency content inherent in the seismic inputs. It is clear that carrying out a stability analysis using the time history method will correctly capture the dynamic properties of the dam system under seismic loading. Having said this, it is also
clear that a sufficient number of time histories are required to ensure the dynamic response of the dam is enveloped.

Overturning for gravity dams under seismic loading is generally not the governing failure mode and is evaluated by acknowledging the cyclic and rapid nature of a seismic event. From the overturning FoS time histories it is clear no overturning will occur for all three cases. This is a different approach when compared to the standard FoS calculation but is more insightful (**).

Damping had a large influence on the resultant cumulative displacement. Adjusting the damping from 5 % to 7 % significantly decreased the final permanent displacement. It is therefore imperative that the appropriate damping be used for the relevant load case considered.

### Table 2. Stability results

<table>
<thead>
<tr>
<th>Check</th>
<th>Seismic Coeff</th>
<th>Response Spectrum</th>
<th>SANS 5%</th>
<th>SANS 7%</th>
<th>1940 El Centro 5%</th>
<th>1940 El Centro 7%</th>
<th>1979 El Centro 5%</th>
<th>1979 El Centro 7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>FoS Overtorn</td>
<td>1.41</td>
<td>0.70</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>FoS Sliding</td>
<td>1.01</td>
<td>0.80</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Displace (mm)</td>
<td>*</td>
<td>*</td>
<td>1.26</td>
<td>0.38</td>
<td>72.50</td>
<td>38.10</td>
<td>8.45</td>
<td>1.20</td>
</tr>
</tbody>
</table>

5. **SIMPLIFIED APPROACH**

From the above information, it is clear that there is a need to evaluate seismic stability using linear time history analysis. It is common practice that, for early design stage, the assessment of a gravity dam stability may be carried out using pseudostatic methods. However, this method of early stage assessment does not correctly capture the dynamic behaviour the dam structure and can produce misleading results.

It is then of worth to establish a simplified model which can be used in early stage design but also sufficiently captures the dynamic behaviour of the dam under seismic loading. To enable the simplified assessment, it is necessary to convert the detailed plain strain model to a Single Degree of Freedom (SDOF) system.

### 5.1 Problem idealisation

The simplification of the problem to a SDOF system is possible by assuming that, for stability purposes, the only major contributing mode is the system’s first fundamental frequency. This mode shape is seen to be similar to that of a cantilever. Therefore what is required is to identify the dynamic variables involved in the problem and quantify them to a suitable level of accuracy. Figure 7 shows an illustration of a SDOF system to which the dam will be adapted.

![Figure 7. SDOF Idealisation](image-url)
5.2 Dynamic properties

Reviewing the equation of motion, Equation 1, reveals the relevant variables which need to be addressed to establish an accurate representation of the dam model. Although debatable, the viscous damping is generally defined in the relevant standards and taken as 5% or 7%, dependant on seismic loading conditions.

\[ m \ddot{u} + c \dot{u} + k u = -m \dddot{u}(t) \quad \text{Equation 1} \]

It is therefore only required to determine the mass and stiffness associated to the first fundamental mode which is assumed to be the dominant factor in stability analysis.

5.3 Mass contribution

The normalised fundamental mode shape and displacements were used to calculate the mass to be included in the analysis. It is noted that there are two components of mass to be considered, namely the dam structure and the added mass due to fluid structure interaction. The two mass components are addressed separately.

The contributing mass of the dam structure is calculated based on theory established by Lokke & Chopra (2013). Using the full mass component derived from the first mode and one third the mass derived from the higher modes, to be conservative, the effective mass contribution is given in Equation 2. With \( H_{\text{dam}} \) the height of the dam structure in meters and \( \rho_s \) the density of the structure in kg/m³.

\[ m_{\text{eff}}(y) = \int_0^{H_{\text{dam}}} \frac{\rho_s}{3} \left( 1 + 2 \Gamma_1 \cdot \phi_1(y) \right) dy \quad \text{Equation 2} \]

\[ M_1 = \int_0^{H_{\text{dam}}} \rho_s \cdot \phi_1^2(y) \, dy \quad \text{Equation 3} \]

\[ L_1 = \int_0^{H_{\text{dam}}} \rho_s \cdot \phi_1(y) \, dy \quad \text{Equation 4} \]

\[ \Gamma_1 = \frac{L_1}{M_1} \quad \text{Equation 5} \]

To calculate the normalised component of displacement, the standard mode shape, as given in Figure 8, can be used with little error. It is noted that the integral given in Equations 3 and 4 may be carried out by discretising the dam structure into portions and carrying out numerical integration techniques. For a standard cross section gravity dam, the aforementioned discretisation may be fairly coarse which allows for easy implementation.

![Standardised Mode Shape](image)

**Figure 8.** Normalised standard fundamental mode shape
For this study, it was assumed that the hydrodynamic effects are sufficiently captured using Westergaard (1933) added mass formulation. It should be noted that the calculation of mass contribution from hydrodynamic effects can be varied and the current method adapted. Furthermore, it is assumed that the full hydrodynamic mass be included in the stability analysis. The hydrodynamic added mass contribution is given in Equation 6 and subsequent total mass contribution given in Equation 7.

\[
m_{\text{add}} = 0.583 \cdot \rho_w \cdot H_w^2 \tag{Equation 6}
\]

\[
m_t = m_{\text{eff}} + m_{\text{add}} \tag{Equation 7}
\]

### 5.4 Stiffness calculation

The first fundamental period of a standard dam cross section on rigid foundation is given in Equation 8, Chopra (1978), with \( E_{\text{dam}} \) the elastic modulus of the dam in MPa.

\[
T_0 = 0.381 \cdot \frac{H_{\text{dam}}}{\sqrt{E_{\text{dam}}}} \tag{Equation 8}
\]

The equation used to calculate \( T_0 \) does not account for the dam foundation interaction which, in general, increases the fundamental period of the dam empty. A parametric study on the Pine Flat model was carried out and values established to allow for the consideration of dam foundation interaction. The period lengthening factors, dependant on the stiffness of the dam and foundation, are shown in Table 3. The values may be linearly interpolated with small error. The fundamental period including dam foundation interaction is given in Equation 9.

\[
T_1 = T_0 \cdot \psi \tag{Equation 9}
\]

<table>
<thead>
<tr>
<th>Foundation Stiffness (GPa)</th>
<th>Dam 15GPa</th>
<th>Dam 20GPa</th>
<th>Dam 25GPa</th>
<th>Dam 30GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>2.20</td>
<td>2.15</td>
<td>2.12</td>
<td>1.97</td>
</tr>
<tr>
<td>5.00</td>
<td>1.70</td>
<td>1.63</td>
<td>1.59</td>
<td>1.56</td>
</tr>
<tr>
<td>10.00</td>
<td>1.38</td>
<td>1.29</td>
<td>1.24</td>
<td>1.20</td>
</tr>
<tr>
<td>15.00</td>
<td>1.26</td>
<td>1.16</td>
<td>1.10</td>
<td>1.05</td>
</tr>
<tr>
<td>20.00</td>
<td>1.19</td>
<td>1.09</td>
<td>1.02</td>
<td>0.97</td>
</tr>
<tr>
<td>30.00</td>
<td>1.13</td>
<td>1.01</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>60.00</td>
<td>1.06</td>
<td>0.94</td>
<td>0.86</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Using \( T_1 \) and the effective mass of the dam structure, the stiffness value of the dam for the fundamental mode assuming a SDOF system is given in Equation 10.

\[
k = \left( \frac{2\pi}{T_1} \right)^2 \cdot m_{\text{eff}} \tag{Equation 10}
\]

Now having the stiffness of the system \( (k) \) and the total mass \( (m_t) \) a simple spring-mass model can be established, with viscous damping defined as per relevant standards or literature. It is noted that no further adjustment is made for reservoir-dam interaction.

### 6. SIMPLIFIED APPROACH CASE STUDY

To evaluate the effectiveness of the established simplified model, two cases were reviewed. The first is that of the synthetic time history as per SANS, and the second analysed under the El Centro 1940 time history. Both analyses assumed a constant 5 % modal damping.
6.1 Results

To post process results, it is recognised that the base shear is simply equal to the reaction force of the system. The overturning moment comprises two components, that of the dam structure and that of the hydrodynamic mass. The overturning moment is calculated by multiplying the reaction force by the percentage mass contribution and lever arm of each of the components and taking the sum of the two.

The cumulative displacement result is, as before, calculated using the double integral method. The FoS time histories for the SDOF system and associated MDOF finite element system are shown in Figure 9. It is clear that the SDOF system sufficiently captures the frequency dependant dynamic behaviour of the dam system. The magnitudes of the two models remain meaningfully similar.

![Figure 9. Sliding and overturning FoS time histories for SDOF and FEM](image)

Figure 10 gives a comparative of the cumulative displacements of the SDOF system to that of the MDOF finite element system. Due to the above mentioned conservative assumptions, the SDOF system over predicts the cumulative displacements. This is a welcomed result as for early design stage, a slightly more conservative outcome will provide a safer design without being excessively over conservative leading to uneconomical designs.

![Figure 10. Cumulative displacement comparison, SDOF and FEM](image)

7. CONCLUSIONS

The stability analysis of a gravity dam is a crucial design check. A theoretically correct method is required to address the complex nature of seismic events. Established methods allow for more simple approaches to be used in early stage design. These methods may not be capable of capturing the dynamic behaviour of the dam system and its response to seismic loading.

All three components of dynamic behaviour need to be addressed namely mass, damping and stiffness. These components are not at all addressed in the seismic coefficient approach where stability checks may either be over or under conservative dependant on factors included due to analysis type or structural importance. These factors are decided upon by the design engineer, but have no real theoretical background with regards to dynamics. It is therefore clear that this method is not appropriate for stability analysis.
Dynamic properties of the system are incorporated in methods such as the response spectrum method. The difficulty arises due to the fact that peak responses from a SDOF system are utilised to develop the input response spectrum. Over conservatism is therefore inherent in the method and may lead to costly designs if used in stability analysis.

There is a clear need to evaluate stability using time history analysis. This will allow for a deeper understanding of the dynamic behaviour of the dam. Lacking site specific information, at least three carefully selected time histories are required to correctly envelope the dynamic response of the dam system. A permanent cumulative displacement based criteria may be used to determine sliding stability. The question is what the allowable cumulative displacement should be. Firstly this depends on the dam type. For earth dams it has be suggested that, excluding liquefaction concerns, 600 mm may be acceptable, FEMA (2005). For concrete dams it is suggested that 50 mm may be acceptable, Fenves & Chavez (1996). Ultimately this is dependent on post seismic stability, operational requirements and magnitude of seismic event. Further research needs to be carried out to determine suitable permanent displacement criteria.

A detailed model may not be required for early stage design where traditional methods could have been used. This paper proposes a SDOF system which captures the dynamic behaviour of the dam system. The results are seen as fairly conservative, but not to the extent where they would produce an uneconomical dam design. As the prosed method is a SDOF system, complex matrix algebra is not required and the solution may be carried out in excel, VBA, using direct integration schemes including Wilson θ Method or Newmark Method. The results can then be included in the static calculation and final stability results obtained.

8. REFERENCES


